

# FIJI MATHEMATICS ASSOCIATION



## FIJI MATHEMATICS COMPETITION (FMC) YEAR 13

**Thursday 1<sup>st</sup> September 2016**

**Time Allowed: 1 Hour 15 minutes**

**Note:**

**Calculators are NOT permitted.**

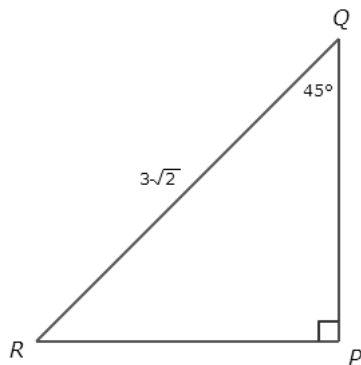
**Diagrams are NOT drawn to scale.**

**Instructions:**

1. Print your **Name** in the space provided and Shade the circle corresponding to your **Year** on the answer sheet.
2. Shade the circle corresponding to your answer with dark pencil on the answer sheet provided.
3. Multiple answers **will not be** accepted.

- The value of  $\lim_{x \rightarrow 1} \frac{3x^2 - x - 2}{2x^2 + x - 3}$  is equal to  
 A. Undefined    B.  $\frac{0}{0}$     C.  $\frac{3}{2}$     D. 1    E.  $\infty$
- Which of the following is factor of  $4x^3 - 15x^2 + 17x - 6$ ?  
 A. 2    B. -1    C. 3    D. -2    E. 0
- Which recursive formula defines this sequence  $(-2, 8, 18, 28, 38, 48, \dots)$  for  $n > 1$ ?  
 A.  $a_n = a_{n-1} - 10$     B.  $a_n = -4a_{n-1}$     C.  $a_n = a_{n-1} + 10$   
 D.  $a_{n+1} = a_n - 10$     E.  $a_n = 10a_{n-1}$

- Find the length PQ.



- $\frac{1}{3}$
- $\sqrt{2}$
- $\frac{1}{3\sqrt{2}}$
- 3
- 1

- Solve for  $x$  in the equation  $4^{x+1} = \frac{1}{64}$ .  
 A. -4    B. -3    C. 2    D. 4    E. 3
- The **vertical** asymptote of the graph of the function  $h(x) = \frac{x^2 - 2x - 3}{2 - x}$  is  
 A. 2    B.  $x = -2$     C.  $x = 2$     D.  $x = \frac{1}{2}$     E.  $y = x$
- If  $x^2yz^3 = 7^4$  and  $xy^2 = 7^5$ , then  $xyz$  equals  
 A. 7    B.  $7^8$     C.  $7^2$     D.  $7^3$     E.  $7^4$
- Evaluate  $(10!/5!)/10$   
 A. 9!    B. 3024    C. 5!    D. 1680    E. 30240
- If  $\log \frac{a}{b} + \log \frac{b}{a} = \log(a+b)$ , then  
 A.  $a-b=1$     B.  $a+b=0$     C.  $a=b$     D.  $a+b=1$     E.  $a^2 - b^2 = 1$
- The sum of all values of  $x$  that satisfy the equation  $(x^2 - 5x + 5)^{x^2 + 3x - 40} = 1$  is  
 A. -3    B. 3    C. -5    D. 8    E. 1

11. In a group of 40 students; 17 own laptop, 19 own desktop computers, and 7 own both. Find the probability that a student picked from this group at random owns neither a laptop nor a desktop computer?
- A.  $\frac{11}{40}$       B.  $\frac{29}{40}$       C.  $\frac{9}{10}$       D.  $\frac{1}{4}$       E.  $\left(\frac{3}{10}\right)$
12. Two trains A and B, 300km apart start towards each other at the same time. They meet after 2 hours. If train B travels  $\frac{8}{7}$  as fast as train A. What is the speed of train A.
- A. 80km/hr      B. 40km/hr      C. 35km/hr      D. 70 km/hr      E. 95.45km/hr
13. It is given that  $w = a + bi$ ,  $a, b \in \mathbb{R}$ . Given also that  $|w| = 4$  and  $\arg w = \frac{5\pi}{6}$ . Find the values of  $a$  and  $b$ .
- A.  $a = 2, b = 2\sqrt{3}$       B.  $a = -2\sqrt{3}, b = 2$       C.  $a = -2, b = 2i$   
 D.  $a = -\frac{\sqrt{3}}{2}, b = \frac{1}{2}$       E.  $a = -2\sqrt{3}, b = 2\sqrt{3}$
14. Two painters paint a house in 3 days. The first painter can finish the house in 4 days if he works alone. If the second painter works alone how many days will he need to finish painting the house?
- A. 4      B. 8      C. 12      D. 10      E. 7
15. Solve  $\cos^2 x + \cos x = \sin^2 x$  on  $0^\circ \leq x \leq 180^\circ$ .
- A.  $x = 60^\circ$       B.  $x = 60^\circ, 180^\circ$       C.  $x = 180^\circ$       D.  $x = 0^\circ$       E.  $x = 60^\circ, 120^\circ$
16. Find the complex number  $z$  such that  $(4 + 2i)z + (8 - 2i)\bar{z} = -2 + 10i$ .
- A.  $1 - \frac{5}{2}i$       B.  $-1 - \frac{5}{2}i$       C.  $-1 + \frac{5}{2}i$       D.  $1 + \frac{5}{2}i$       E.  $-\frac{5}{2} - i$
17. If  $y = e^{\cos 2x}$ , then  $\frac{dy}{dx}$  is:
- A.  $-\cos 2x e^{\sin 2x}$       B.  $\cos 2x e^{\sin 2x}$       C.  $2 e^{\sin 2x}$   
 D.  $-2 \sin 2x e^{\cos 2x}$       E.  $2 \sin 2x e^{\cos 2x}$
18. Which of the following is the upper bound of the sequence  $a_n = \frac{2n+4}{4n+3}$
- A.  $\frac{8}{9}$       B.  $\frac{4}{3}$       C.  $\frac{9}{10}$       D.  $\frac{6}{7}$       E.  $\frac{1}{2}$
19. Determine the area of the region enclosed by  $x = \sqrt{y}$  and  $y = \sqrt{x}$ .
- A.  $\frac{1}{3}$       B. 0      C. 1      D.  $-\frac{1}{3}$       E. -1
20. Evaluate  $\sum_{i=1}^{50} [\ln(i+3) - \ln(i+2)]$ .
- A.  $\ln 53 + \ln 3$       B.  $\ln\left(\frac{53}{3}\right)$       C.  $\ln 159$       D.  $\ln 50$       E.  $\ln\left(\frac{3}{53}\right)$

21. Find the coordinates of the point P where P lies on the line AB such that:  $A=(4,-3,1)$ ,  $B=(-1,2,-4)$  and  $AP:PB=3:2$ .
- A.  $(5,2,3)$       B.  $(1,0,-2)$       C.  $(-1,0,2)$       D.  $(2,-1,-1)$       E.  $(1,-1,2)$
22. If  $f(x) = (2\sqrt{x}+1)\left(\frac{2-x}{x^2+3x}\right)$ , find  $f'(1)$ .
- A.  $\frac{3}{4}$       B. 1      C.  $\frac{23}{16}$       D.  $\frac{-23}{8}$       E.  $\frac{-23}{16}$
23. Find the absolute maximum values of  $f(x) = \sqrt[3]{x}(8-x)$  on the interval  $[0,8]$ .
- A. 0      B.  $6\sqrt[3]{2}$       C. 8      D. 6      E. 4
24. Find all the values of  $x^*$  in the interval  $[-12,0]$  that satisfy the equation  $\int_a^b f(x)dx = f(x^*)(b-a)$ , where  $f(x) = x^2 + x$ .
- A.  $x^* = -7$       B.  $x^* = -7,6$       C.  $x^* = -6$       D.  $x^* = 6,7$       E.  $x^* = -6,-7$
25. What is the 3<sup>rd</sup> term in the expansion of  $(7y-5z)^4$  (in decreasing powers of y).
- A.  $-3500yz^3$       B.  $6860y^3z$       C.  $-7350y^2z^2$       D.  $3500y^2z^2$       E.  $7350y^2z^2$
26. Find the value(s) of  $k$  such that  $g(x) = \begin{cases} kx^2 + 2 & \text{if } x \geq 2 \\ -2x + 2 & \text{if } x < 2 \end{cases}$  is continuous?
- A.  $k=1$       B.  $k=-1$       C.  $k=2$       D.  $k=0$       E.  $k=-2$
27. Find the volume of the solid that results when the region enclosed by the curve  $y=9-x^2$  and line  $y=0$  is revolved about the  $x$ -axis.
- A.  $\frac{1296}{5}$       B.  $\frac{648\pi}{5}$       C.  $1296\pi$       D.  $\frac{1296\pi}{5}$       E.  $\frac{648}{5}$
28. If  $f(x) = px + q$  and  $f(f(f(x))) = 8x + 21$ , and if  $p$  and  $q$  are real numbers, then what is the value of  $p + q$ ?
- A. 5      B. 10      C. 3      D. 2      E. 1
29. Air is being pumped into a spherical balloon at a rate of  $5\text{cm}^3/\text{min}$ . Determine the rate at which the radius of the balloon is increasing when the diameter of the balloon is 20 cm.
- A.  $10\text{cm}/\text{min}$       B.  $\frac{1}{80\pi}\text{cm}/\text{min}$       C.  $\frac{1}{5\pi}\text{cm}/\text{min}$       D.  $\frac{1}{320\pi}\text{cm}/\text{min}$       E.  $5\text{cm}/\text{min}$
30. Find an equation of the line tangent to the graph of  $(x^2 + y^2)^3 = 8x^2y^2$  at the point  $(-1,1)$ .
- A.  $y = x - 2$       B.  $x = y - 2$       C.  $y = 2 - x$       D.  $y = x$       E.  $y = 2$